Enrollment No: _____Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2022

Subject Name: Graph Theory

Subject Code: 4SC06GRT1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 23/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Define: Regular graph (02)

b) Draw a diagraph with 7 vertices in which three vertices have 2 degree, two vertices have 4 degree and remaining vertices have 6 degree. (02)

c) Define: Isomorphism of two graphs (02)

d) State Dirac's theorem. (02)

e) Define: Connected graph (02)

f) True/False: K_6 is a Euler graph. (01)

g) Is C_n a Hamiltonian graph? (01)

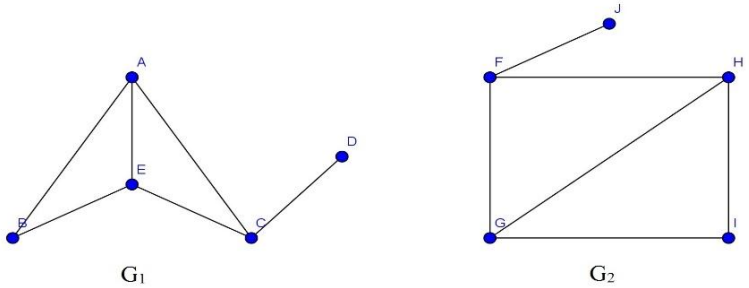
h) How many edges in K_5? (01)

i) True/False: Incidence matrix is a symmetric matrix. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Show that the following graphs are isomorphic. (07)



G_1 G_2

Figure – 1

b) State and prove First Theorem of Graph Theory and also verify it for K_3. (05)

c) Define Degree Sequence (02)

Q-3 Attempt all questions **(14)**

a) In a complete graph with n vertices, there is $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits if $n > 3$ and n is also an odd. (05)

b) Prove that every tree has either one or two centers. (05)

c) Answer the following for graph which shows in figure-2. (04)

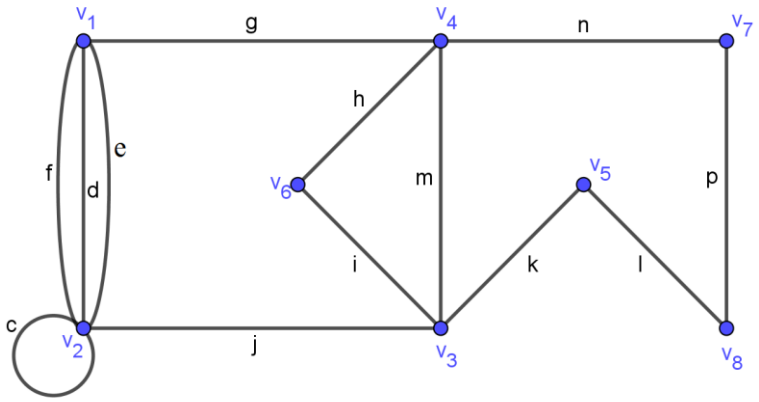


Figure – 2

i) Write degree of each vertex.
ii) How many odd and even vertices?
iii) Write one path of length 7.
iv) Write one closed walk of length 10.

Q-4 Attempt all questions **(14)**

a) State and prove necessary and sufficient condition for the graph is disconnected. (07)

b) State and prove Euler's theorem. (07)

Q-5 Attempt all questions **(14)**

a) Let G be a simple graph with n vertices and k-components then G have at most (07)

b) Find center, radius and diameter of the following graph. (05)

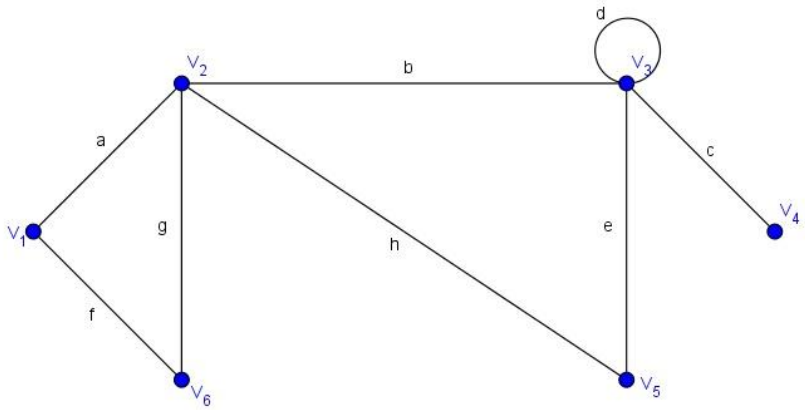


Figure – 3

c) Define: Hamiltonian circuit (02)

Q-6 Attempt all questions **(14)**

a) Explain Konigsberg bridge problem and write the solution given by Euler and also define Euler graph. (07)

b) Define: Tree and prove that G has $(n-1)$ edges, If G be a tree with n is vertices. (07)

Q-7 Attempt all questions **(14)**

a) Answer the following questions from the figure-4: (07)

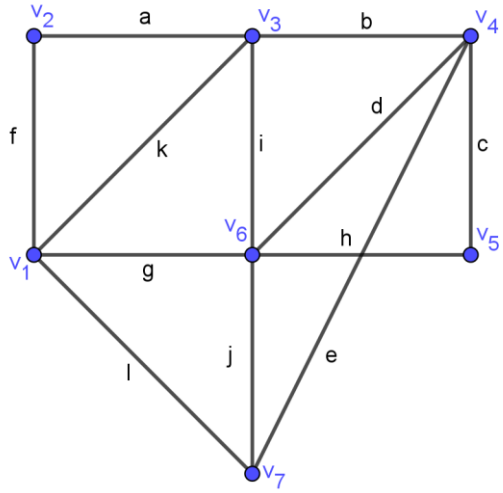


Figure-4

i) Write one spanning tree.

ii) Write three fundamental cut-sets w.r.t. i).

iii) Write one fundamental circuit w.r.t. i).

iv) How many branches and chords are in this graph?

v) What is the vertex and edge connectivity of this graph?

b) Define: Connected graph, Spanning subgraph, Cycle, Spanning tree (04)

c) Find the adjacency matrix for the following figure-5: (03)

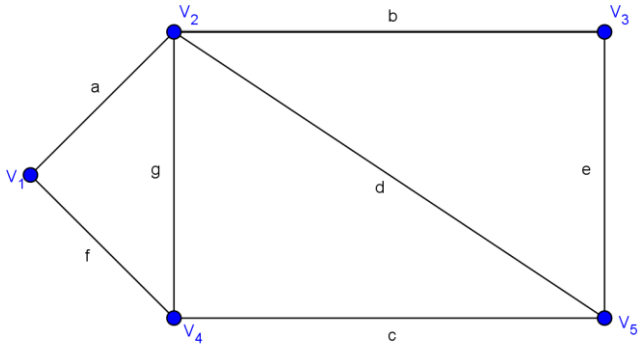


Figure – 5

Q-8 Attempt all questions **(14)**

a) Verify $AB^T = O$ for figure-5, where A&B are incidence & circuit matrix respectively. (07)

b) Find the path matrix $P(V_2, V_5)$ for figure-5. (05)

c) Define: Cut-set matrix (02)